**Prerequisite Information**

* Properties of Random Variables
  + If and are two continuous random variables described by probability density functions and respectively, then the joint distribution is defined as follows.
  + If and are three continuous random variables described by probability density functions and respectively, then the following property holds

where is the set of all possible values of the variable . This leads to the next property.

* + If , , and are three continuous random variables described by probability density functions , , and respectively, then the following property holds
* Ratio Distribution density function
  + If and are two continuous random variables described by probability density functions and respectively, then the ratio distribution defined as has a density function that can be calculated through the following formula. (See Proof)

**Proof of Density function for the Yield variable**

In the current model of the Yield, the random variable is given by the following expression

where , , and are constants and , , and are independent normally distributed variables distributed as , , and . Thus, the known density functions are , , and .

A variable switch was introduced where and . Thus,

Using the Ratio distribution density function formula gives the distribution of the Yield as

Now, the joint distribution must be calculated. Using the third listed property of random variables, we have that

In order to calculate the joint distribution , we use the first listed property of random variables, where

Then, because and are independent without , the conditional joint distribution can be calculated as

The remaining conditional variables are then

Then, the probability density functions to plug in are

Thus, summarizing previous steps

Evaluating this expression gives the result of

Where